Chapter 2 Quadrilaterals

VERY SHORT ANSWER TYPE QUESTIONS

Question.1 Three angles of a quadrilateral are equal and the fourth angle is equal to 144°. Find each of the equal angles of the quadrilateral.

Solution.

Let each equal angle of given quadrilateral be x.

We know that, sum of all interior angles of a quadrilateral is 360°.

$\therefore \quad x + x + x + 144° = 360°$

$3x = 360° - 144°$

$3x = 216°$

$x = 72°$

Hence, each equal angle of the quadrilateral is of 72° measures.

Question.2 Two consecutive angles of a parallelogram are (x + 60)° and (2x + 30)°. What special name can you give to this parallelogram ?

Solution.

We know that consecutive interior angles of a parallelogram are supplementary.

$\therefore (x + 60)° + (2x + 30)° = 180°$

$\Rightarrow \quad 3x° + 90° = 180°$

$\Rightarrow \quad 3x° = 90°$

$\Rightarrow \quad x° = 30°$

Thus, two consecutive angles are $(30 + 60)°$, $(2 \times 30 + 30)°$ *i.e.*, 90° and 90°.

Hence, the special name of the given parallelogram is rectangle.

Question.3 If one angle of a parallelogram is 30° less than twice the smallest angle, then find the measure of each angle.

Solution.

Let smallest angle be x

One angle $= 2x - 30°$

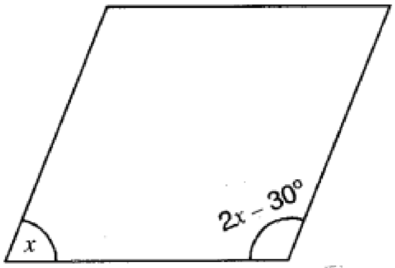
$\therefore x + 2x - 30° = 180°$

$\Rightarrow \quad 3x = 210°$

$\Rightarrow \quad x = 70°$

$\therefore \quad 2x - 30° = 110°$

$\therefore$ Angles are 70°, 110°, 70°, 110°



Question.4 If one angle of a parallelogram is twice of its adjacent angle, find the angles of the parallelogram. [CBSE-15-6DWMW5A]

Solution.

Let the two adjacent angles be $x°$ and $2x°$.

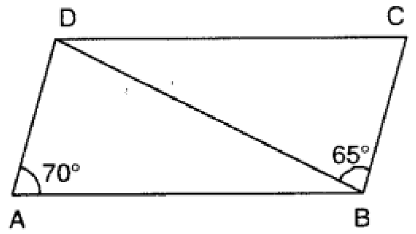
In a parallelogram, sum of the adjacent angles are $180°$.

$$\therefore \quad x + 2x = 180°$$
$$\Rightarrow \quad 3x = 180°$$
$$\Rightarrow \quad x = 60°$$

Thus, the two adjacent angles are $120°$ and $60°$. Hence, the angles of the parallelogram are $120°$, $60°$, $120°$ and $60°$.

Question.5

In the given figure, ABCD is a parallelogram in which $\angle DAB = 70°$ and $\angle DBC = 65°$, then find the measure of $\angle CDB$.



Solution.

$$\angle A + \angle B = 180°$$
$$\Rightarrow \quad 70° + \angle B = 180°$$
$$\Rightarrow \quad \angle B = 110°$$
$$\Rightarrow \quad \angle ABD + \angle DBC = 110°$$
$$\Rightarrow \quad \angle ABD + 65° = 110°$$
$$\Rightarrow \quad \angle ABD = 45°$$
$$\text{Now,} \quad \angle CDB = \angle ABD = 45°$$

Question.6.If the diagonals of a quadrilateral bisect each other at right angles, then name the quadrilateral.
Solution. Rhombus.

Question.7 In quadrilateral PQRS, if $\angle P = 60°$ and $\angle Q : \angle R : \angle S = 2:3:7$, then find the measure of $\angle S$.

Solution.

Let $\angle Q = 2x$, $\angle R = 3x$ and $\angle S = 7x$

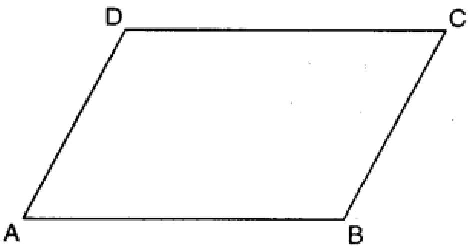
Now, $\angle P + \angle Q + \angle R + \angle S = 360°$

$$\Rightarrow \quad 60° + 2x + 3x + 7x = 360°$$
$$\Rightarrow \quad 12x = 300°$$
$$\Rightarrow \quad x = \frac{300°}{12} = 25°$$
$$\angle S = 7x = 7 \times 25° = 175°$$

Question.8 If an angle of a parallelogram is two-third of its adjacent angle, then find the smallest angle of the parallelogram.

Solution.

In a parallelogram ABCD,



Let $\angle A$ be x and $\angle B$ be $\frac{2}{3}x$

$\therefore \quad \angle A + \angle B = 180^\circ$

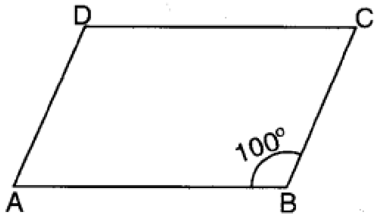
$\Rightarrow \quad x + \frac{2}{3}x = 180^\circ$

$\Rightarrow \quad \frac{5}{3}x = 180^\circ$

$\Rightarrow \quad x^\circ = 180^\circ \times \frac{3}{5} = 108^\circ$

$\angle A = 108^\circ, \angle B = \frac{2}{3} \times 108^\circ = 72^\circ$

Question.9 In the given figure, ABCD is a parallelogram. If ∠B = 100°, then find the value of ∠A +∠C.



[opp. ∠s of a ||gm]

Solution.

$\because$ ABCD is a parallelogram

$\therefore \quad \angle A + \angle B = 180^\circ$

$\Rightarrow \quad \angle A + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle A = 80^\circ$

$\because \quad \angle C = \angle A = 80^\circ$

$\therefore \quad \angle A + \angle C = 80^\circ + 80^\circ$

$= 160^\circ$

Question.10 If the diagonals of a parallelogram are equal, then state its name.

Solution. Rectangle

Question.11 ONKA is a square with ∠KON = 45°. Determine ∠KOA.

Solution.

Since ONKA is a square

$\therefore \quad \angle AON = 90^\circ$

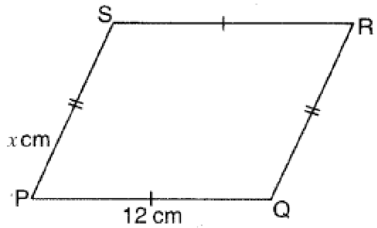
We know that diagonal of a square bisects its ∠s

$\Rightarrow \quad \angle AOK = \angle KON = 45^\circ$

Hence, $\quad \angle KOA = 45^\circ$

Question.12 PQRS is a parallelogram, in which PQ = 12 cm and its perimeter is 40 cm. Find the length of each side of the parallelogram.

Solution.



Here, PQ = SR = 12 cm

Let PS = x and PS = QR

$\therefore\ x + 12 + x + 12 =$ Perimeter

$$2x + 24 = 40$$

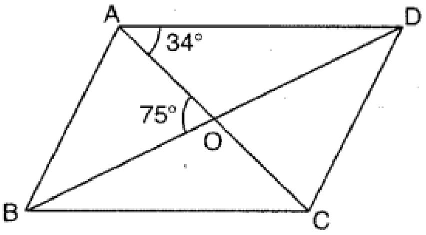
$$2x = 16$$

$$x = 8$$

Hence, length of each side of the parallelogram is 12 cm, 8 cm, 12 cm and 8 cm.

Question.13

The diagonals AC and BD of parallelogram ABCD intersect at the point O. If $\angle DAC = 34°$ and $\angle AOB = 75°$, then what is the measure of $\angle DBC$?



Solution.

ABCD is a parallelogram.

$\therefore$ $AD \parallel BC \Rightarrow \angle ACB = \angle DAC = 34°$

Now, $\angle AOB$ is an exterior angle of ΔBOC

$\therefore$ $\angle OBC + \angle OCB = \angle AOB$ [$\because$ ext $\angle$ = sum of two int. opp. $\angle s$]

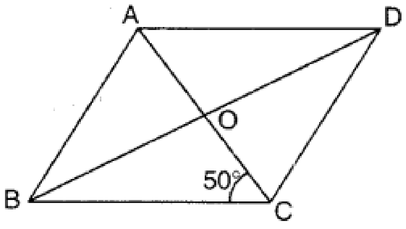
$\Rightarrow$ $\angle OBC + 34° = 75°$

$\Rightarrow$ $\angle OBC = 75° - 34° = 41°$

or $\angle DBC = 41°$

Question.14

In figure, ABCD is a rhombus such that $\angle ACB = 50°$, then what is the measure of $\angle ADB$?



Solution.

AD || BC ∴ $\angle CAD = \angle ACB = 50°$ [alt. int. $\angle s$]

∵ The diagonals of a rhombus are at right angle to each other.

∴ $\angle AOD = 90°$

Now, in ΔAOD, we have

$$\angle ADO + \angle DAO + \angle AOD = 180°$$

$$\Rightarrow \quad \angle ADO + 50° + 90° = 180°$$

$$\Rightarrow \quad \angle ADO + 140° = 180°$$

$$\Rightarrow \quad \angle ADO \text{ or } \angle ADB = 40°$$

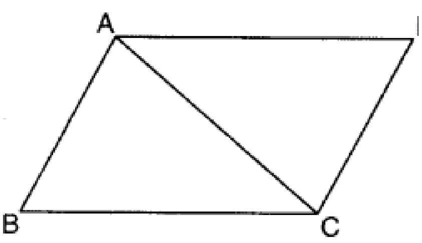
Question. 15.If ABCD is a parallelogram, then what is the measure of ∠A – ∠C ?

Solution. ∠A −∠C = 0° [opposite angles of parallelogram are equal]

Short Answer Questions Type-1

Question.16 Prove that a diagonal of a parallelogram divide it into two congruent triangles. [CBSE March 2012]

Solution. Given : A parallelogram ABCD and AC is its diagonal.



To Prove : $\Delta ABC \cong \Delta CDA$

Proof : In ΔABC and ΔCDA, we have

$\angle DAC = \angle BCA$ [alt. int. angles, since AD || BC]

$AC = AC$ [common side]

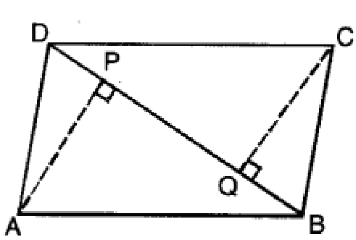
and $\angle BAC = \angle DCA$ [alt. int. angles, since AB || DC]

∴ By ASA congruence axiom, we have

$\Delta ABC \cong \Delta CDA$

Question.17 ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see fig.). Show that :
(i) AAPB ≅ ACQD (ii) AP = CQ [CBSE March 2012]

Solution.



Given : In ||gm ABCD, AP and CQ are perpendiculars from the vertices A and C on the diagonal BD.

To Prove : (i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Proof : (i) In ΔAPB and ΔCQD

$AD = BC$ [opp. sides of a ||gm ABCD]

$\angle APB = \angle DQC$ [each = 90°]

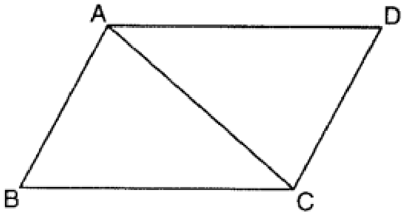
$\angle ABP = \angle CDQ$ [alt. int. $\angle s$]

$\Rightarrow \Delta APB \cong \Delta CQD$ [by AAS congruence axiom]

(ii) $\Rightarrow$ $AP = CQ$ [c.p.c.t.]

Question.18

ABCD is a rhombus with $\angle ABC = 50°$. Determine $\angle ACD$. ***[CBSE March 2012]***



Solution.

Since ABCD is a rhombus.

$\Rightarrow$ ABCD is also a parallelogram.

$\therefore \quad \angle ADC = \angle ABC = 50°$ [opp. angles of a parallelogram are equal]

$\because$ Adjacent sides of a rhombus are equal

$\therefore \quad AD = DC$

$\Rightarrow \quad \angle DAC = \angle ACD$ [angles opposite to equal sides are equal]

$= x$ (say)

$\therefore$ In ΔACD, we have

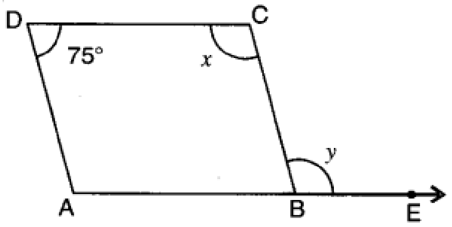
$\angle DAC + \angle ACD + \angle ADC = 180° \quad \Rightarrow \quad x + x + 50° = 180°$

$\Rightarrow \quad 2x = 130° \quad \Rightarrow \quad x = 65°$

$\therefore \quad \angle ACD = 65°$

Question.19

ABCD is a parallelogram in which $\angle ADC = 75°$ and side AB is produced to point E as shown in the figure. Find $x + y$. ***[CBSE-14-ERFKZ8H]***



Solution.

Here, $\angle C$ and $\angle D$ are adjacent angles of the parallelogram.

$\therefore \quad \angle C + \angle D = 180°$

$\Rightarrow \quad x + 75° = 180°$

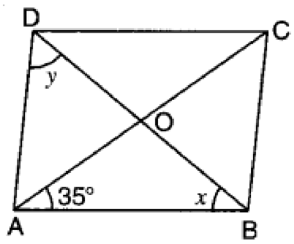
$\Rightarrow \quad x = 105°$

Also, $y = x = 105°$ [alt. int. angles]

Thus, $x + y = 105° + 105° = 210°$

Question.20

In the figure, ABCD is a rhombus, whose diagonals meet at O. Find the values of x and y. ***[CBSE-14-17DIG1U]***



Solution.

Since diagonals of a rhombus bisect each other at right angle.

$\therefore$ In ΔAOB, we have

$$\angle OAB + \angle x + 90^\circ = 180^\circ$$

$$\angle x = 180^\circ - 90^\circ - 35^\circ \quad [\because \angle OAB = 35^\circ]$$

$$= 55^\circ$$

Also, $\angle DAO = \angle BAO = 35^\circ$

$\therefore \quad \angle y + \angle DAO + \angle BAO + \angle x = 180^\circ$

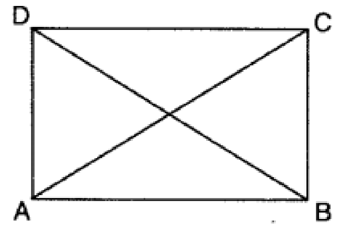
$\Rightarrow \quad \angle y + 35^\circ + 35^\circ + 55^\circ = 180^\circ$

$\Rightarrow \quad \angle y = 180^\circ - 125^\circ = 55^\circ$

Hence, the values of x amd y are $x = 55^\circ$, $y = 55^\circ$.

Question.21 If the diagonals of a parallelogram are equal, then show that it is a rectangle. [CBSE March 2012]

Solution.



Given : A parallelogram ABCD, in which AC = BD.

To Prove : ABCD is a rectangle.

Proof : In ΔABC and ΔBAD

AB = AB [common]

AC = BD [given]

BC = AD [opp. sides of a $||^{gm}$]

$\Rightarrow \quad \Delta ABC \cong \Delta BAD$ [by SSS congruence axiom]

$\Rightarrow \quad \angle ABC = \angle BAD$ [c.p.c.t.]

Also, $\angle ABC + \angle BAD = 180^\circ$ [co-interior angles]

$\Rightarrow \quad \angle ABC + \angle ABC = 180^\circ$ [$\because \angle ABC = \angle BAD$]

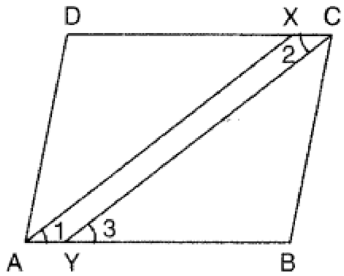
$\Rightarrow \quad 2\angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = \frac{1}{2} \times 180^\circ = 90^\circ$

Hence, parallelogram ABCD is a rectangle.

Question.22 ABCD is a parallelogram and line segments AX, CY bisect the angles A and C, respectively. Show that AX\\CY. D x c

Solution.



Since opposite angles are equal in a parallelogram. Therefore, in parallelogram ABCD, we have

$$\angle A = \angle C$$

$$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle C$$

$$\Rightarrow \quad \angle 1 = \angle 2 \quad ...(i)$$

[$\because$ AX and CY are bisectors of $\angle A$ and $\angle C$ respectively]

Now, AB||DC and the transversal CY intersects them.

$\therefore \quad \angle 2 = \angle 3 \quad ...(ii)$ [$\because$ alternate interior angles are equal]

From (*i*) and (*ii*), we have

$$\angle 1 = \angle 3$$

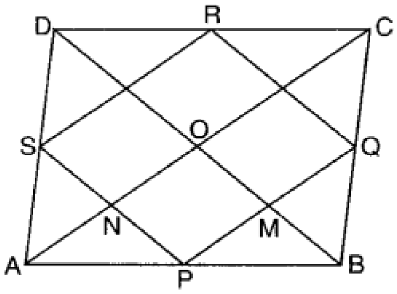
Thus, transversal AB intersects AX and YC at A and Y such that $\angle 1 = \angle 3$ *i.e.*, corresponding angles are equal.

$\therefore \quad AX \parallel CY$

Short Answer Questions Type-II

Question.23

The diagonals of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral formed by joining the mid-points of its sides is a rectangle. [CBSE March 2012]



Solution.

Given : A quadrilateral ABCD whose diagonals AC and BD are perpendicular to each other at O. P, Q, R and S are mid-points of side AB, BC, CD and DA respectively are joined are formed quadrilateral PQRS.

To Prove : PQRS is a rectangle.

Proof : In ΔABC, P and Q are mid-points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ... (*i*) [mid-point theorem]

Further, in ΔACD, R and S are mid-points of CD and DA respectively.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2} AC$ ... (*ii*) [mid-point theorem]

From (*i*) and (*ii*), we have $PQ \parallel SR$ and $PQ = SR$

Thus, one pair of opposite sides of quadrilateral PQRS are parallel and equal.

$\therefore$ PQRS is a parallelogram.

Since $PQ \parallel AC \Rightarrow PM \parallel NO$

In ΔABD, P and S are mid-points of AB and AD respectively.

$\therefore$ $PS \parallel BD$ [mid-point theorem]

$\Rightarrow$ $PN \parallel MO$

$\therefore$ Opposite sides of quadrilateral PMON are parallel.

$\therefore$ PMON is a parallelogram.

$\therefore$ $\angle MPN = \angle MON$ [opposite angles of $\parallel^{gm}$ are equal]

But $\angle MON = 90^\circ$ [given]

$\therefore$ $\angle MPN = 90^\circ \Rightarrow \angle QPS = 90^\circ$

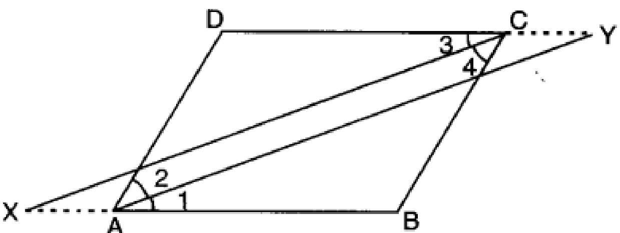
Thus, PQRS is a parallelogram whose one angle is 90°.

$\therefore$ PQRS is a rectangle.

Question.24 ABCD is a quadrilateral in which the bisectors of $\angle A$ and $\angle C$ meet DC produced at Y and BA produced at X respectively. Prove that : [CBSE-15-6DWMW5A]

Solution.

$$\angle X + \angle Y = \frac{1}{2}(\angle A + \angle C)$$



Here, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

In ΔXBC, we have

$$\angle X + \angle B + \angle 4 = 180^\circ$$

$$\angle X + \angle B + \frac{1}{2}\angle C = 180^\circ \quad ...(i)$$

In ΔADY, we have

$$\angle 2 + \angle D + \angle Y = 180^\circ$$

$$\frac{1}{2}\angle A + \angle D + \angle Y = 180^\circ \quad ...(ii)$$

Adding (*i*) and (*ii*), we have

$$\angle X + \angle Y + \angle B + \angle D + \frac{1}{2}\angle C + \frac{1}{2}\angle A = 360^\circ$$

Also, in quadrilateral ABCD,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\therefore \angle X + \angle Y + \angle B + \angle D + \frac{1}{2}\angle C + \frac{1}{2}\angle A = \angle A + \angle B + \angle C + \angle D$$

$$\angle X + \angle Y = \angle A - \frac{1}{2}\angle A + \angle C - \frac{1}{2}\angle C$$

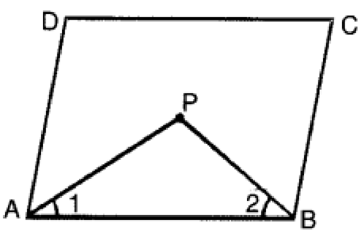
$$\angle X + \angle Y = \frac{1}{2}(\angle A + \angle C)$$

Question.25 In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles. [CBSE March 2012]

Solution.

Given : A parallelogram ABCD such that the bisectors of adjacent angles A and B intersect at P.

To Prove : $\angle APB = 90^\circ$



Proof : Since ABCD is a $||^{gm}$

$\therefore$ AD || BC

$\Rightarrow \angle A + \angle B = 180^\circ$ [sum of consecutive int. $\angle$s]

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$ ...(*i*)

[$\because$ AP is the bisector of $\angle A$ and BP is the bisector of $\angle B$]

$\therefore \angle 1 = \frac{1}{2}\angle A$ and $\angle 2 = \frac{1}{2}\angle B$]

Now, in ΔAPB, we have

$\angle 1 + \angle APB + \angle 2 = 180^\circ$ [sum of three angles of a Δ]

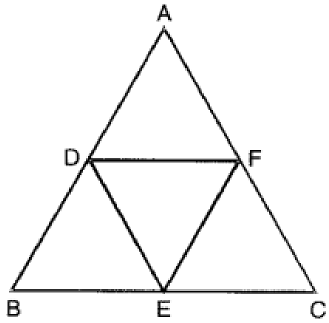
$\Rightarrow 90^\circ + \angle APB = 180^\circ$ [$\because \angle 1 + \angle 2 = 90^\circ$, from (*i*)]

Hence, $\angle APB = 90^\circ$.

Question.26 D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangles ABC is divided into four congruent triangles. [NCERT Exemplar Problem]

Solution.

Since the segment joining the mid-points of any two sides of a triangle is half the third side and parallel to it.



$\therefore$ $DE = \frac{1}{2}AC \Rightarrow DE = AF = CF$

$EF = \frac{1}{2}AB \Rightarrow EF = AD = BD$

$DF = \frac{1}{2}BC \Rightarrow DF = BE = CE$

In ΔDEF and ΔCFE, we have

$DE = CF$ [from (*i*)]

$DF = CE$ [from (*ii*)]

$EF = FE$ [common]

$\Rightarrow$ $\Delta DEF \cong \Delta CFE$ [by SSS criterion of congruence]

Similarly, we have $\Delta DEF \cong \Delta BDE$ and $\Delta DEF \cong \Delta AFD$

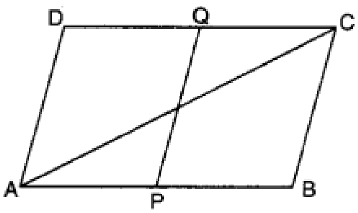
Thus, $\Delta DEF \cong \Delta CFE \cong \Delta BDE \cong \Delta AFD$

Hence, ΔABC is divided into four congruent triangles.

Question.27

Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ (fig.). Show that AC and PQ bisect each other.

[NCERT Exemplar Problem]



Solution.

Join AQ and PC.

Since ABCD is a parallelogram.

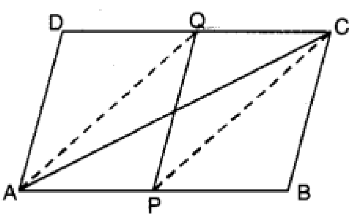
$\Rightarrow$ $AB \parallel DC$

$\Rightarrow$ $AP \parallel QC$

[$\because$ AP and QC are parts of AB and DC respectively]

Also, $AP = CQ$ [given]

Thus, APCQ is a parallelogram.

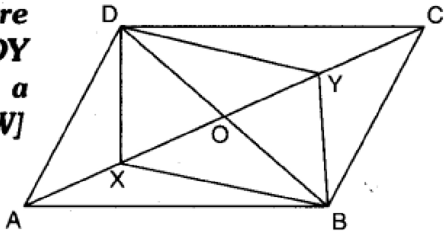


We know that diagonals of a parallelogram bisect each other.

Hence, AC and PQ bisect each other.

Question.28

In quadrilateral ABCD of the given figure, X and Y are points on diagonal AC such that AX = CY and BXDY is a parallelogram. Show that ABCD is a parallelogram. [CBSE-14-GDQNI3W]



Solution.

Since BXDY is a parallelogram.

$\therefore \quad XO = YO \quad \text{.... }(i)$

$DO = BO \quad \text{.... }(ii)$

[$\because$ diagonals of a parallelogram bisect each other]

But $\quad AX = CY \quad \text{.... }(iii)$ [given]

Adding (*i*) and (*iii*), we have

$$XO + AX = YO + CY$$

$\Rightarrow \quad AO = CO \quad \text{.... }(iv)$

From (*ii*) and (*iv*), we have

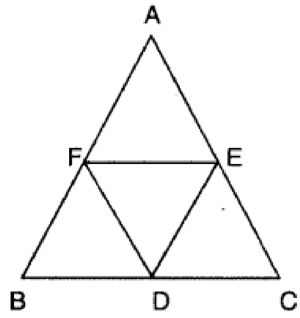
$$AO = CO \text{ and } DO = BO$$

Thus, ABCD is a parallelogram, because diagonals AC and BD bisect each other at O.

Question.29

In the fig., D, E and F are, respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that DEF is also an equilateral triangle.

[CBSE March 2012]



Solution. Since line segment joining the mid-points of two sides of a triangle is half of the third side. Therefore, D and E are mid-points of BC and AC respectively.

$\Rightarrow \quad DE = \frac{1}{2}AB \quad \text{...}(i)$

E and F are the mid-points of AC and AB respectively.

$\therefore \quad EF = \frac{1}{2}BC \quad \text{...}(ii)$

F and D are the mid-points of AB and BC respectively.

$\therefore \quad FD = \frac{1}{2}AC \quad \text{...}(iii)$

Now, ΔABC is an equilateral triangle.

$\Rightarrow \quad AB = BC = CA$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

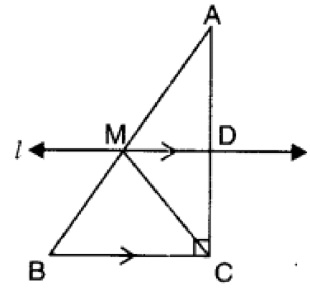
$\Rightarrow \quad DE = EF = FD \quad$ [using (*i*), (*ii*) and (*iii*)]

Hence, DEF is an equilateral triangle.

Long Answer Type Questions

Question.30 ABC is a triangle right-angled at C. A line through the mid-point M of hypotenuse AB parallel to BC intersects AC ad D. Show that:
(i) D is the mid-point of AC
(ii) MD ⊥ AC
(iii) CM = MA = 1/2 AB. [CBSE March 2012]

Solution.



Given : A right ΔABC, right-angled at C. A line through the mid-point M of hypotenuse AB parallel to BC intersects AC at D.

To Prove :

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2} AB$

Proof : (i) Since M is the mid-point of hyp. AB and $MD \parallel BC$.

$\Rightarrow$ D is the mid-point of AC.

(ii) Since $\angle BCA = 90^\circ$

and $MD \parallel BC$ [given]

$\Rightarrow$ $\angle MDA = \angle BCA$

$= 90^\circ$ [corresp. $\angle s$]

$\Rightarrow$ $MD \perp AC$

(iii) Now, in ΔADM and ΔCDM

$MD = MD$ [common]

$\angle MDA = \angle MDC$ [each $= 90^\circ$]

$AD = CD$ [$\because$ D is the mid-point of AC]

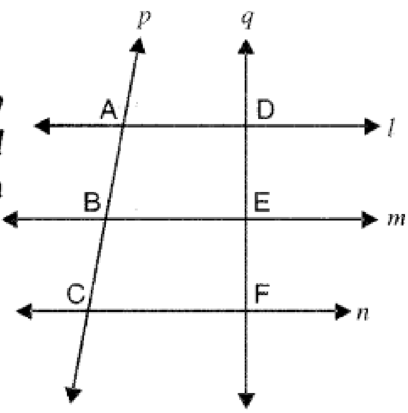
$\Rightarrow$ $\Delta ADM \cong \Delta CDM$ [by SAS congruence axiom]

$\Rightarrow$ $AM = CM$

Also, M is the mid-point of AB [given]

$\Rightarrow$ $CM = MA = \frac{1}{2} AB.$

Question.31

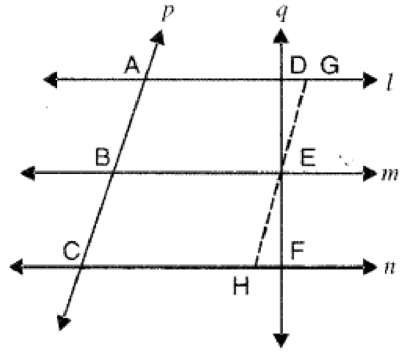


l, m and n are three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts AB and BC on p (see figure). Show that l, m and n cut off equal intercepts DE and EF on q also.

[CBSE March 2012]

Solution.

Through E, draw a line parallel to p intersecting l at G and n at H respectively.



Since $l \parallel m \Rightarrow AG \parallel BE$

and $AB \parallel GE$ [by construction]

$\therefore$ Opposite sides of quadrilateral AGEB are parallel.

$\therefore$ AGEB is a parallelogram.

Similarly, we can prove that BEHC is a parallelogram.

Now, $AB = GE$ [opposite sides of $\|^{gm}$ AGEB]

and $BC = EH$ [opposite sides of $\|^{gm}$ BEHC]

But, given that $AB = BC$. Thus, $GE = EH$

Now, in ΔDEG and ΔFEH, we have

$\angle DEG = \angle FEH$ [vertically opposite angles]

$GE = EH$ [proved above]

and $\angle DGE = \angle FHE$ [alternate interior angles]

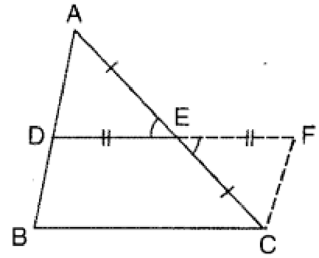
By ASA congruence axiom, we have

$\Delta DEG \cong \Delta FEH$

Hence, $DE = EF$ [c.p.c.t.]

Question.32 The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Solution.



Given : A ΔABC in which D and E are the mid-points of side AB and AC respectively. DE is joined.

To Prove : $DE \parallel BC$ and $DE = \frac{1}{2} BC$.

Const. : Produce the line segment DE to F, such that $DE = EF$. Join FC.

Proof : In Δs AED and CEF, we have

$AE = CE$ [$\because$ E is the mid-point of AC]

$\angle AED = \angle CEF$ [vert. opp. $\angle$s]

and $DE = EF$ [by construction]

$\therefore$ $\Delta AED \cong \Delta CEF$ [by SAS congruence axiom]

$\Rightarrow$ $AD = CF$ ...(*i*) [c.p.c.t.]

and $\angle ADE = \angle CFE$ ...(*ii*) [c.p.c.t.]

Now, D is the mid-point of AB.

$\Rightarrow$ $AD = DB$ ...(*iii*)

From (*i*) and (*iii*), $CF = DB$ ...(*iv*)

Also, from (*ii*)

$\Rightarrow$ $AD \parallel FC$ [if a pair of alt. int. $\angle$s are equal, then lines are parallel]

$\Rightarrow$ $DB \parallel CF$ ...(*v*)

From (*iv*) and (*v*), we find that DBCF is a quadrilateral such that one pair of opposite sides are equal and parallel.

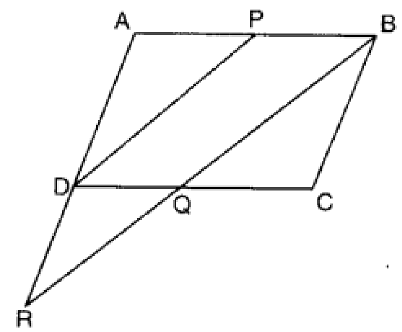
$\therefore$ DBCF is a $\parallel$ gm.

$\Rightarrow$ $DF \parallel BC$ and $DF = BC$ [$\because$ opp. sides of a $\parallel$ gm are equal and parallel]

Also, $DE = EF$ [by construction]

Hence, $DE \parallel BC$ and $DE = \frac{1}{2}BC$. [$\because$ $DE = \frac{1}{2}DF$]

Question.33



P is the mid - point of side AB of a parallelogram ABCD. A line through B parallel to PD meets DC at Q and AD produced at R (see figure). Prove that :

(i) AR = 2BC

(ii) BR = 2BQ ***[CBSE March 2012]***

Solution.

(*i*) In ΔARB, P is a mid-point of AB and $PD \parallel BR$.

$\therefore$ D is a mid-point of AR [converse of mid-point theorem]

$\therefore$ $AR = 2AD$

But $BC = AD$ [opposite sides of $\parallel^{gm}$ ABCD]

Thus, $AR = 2BC$

(*ii*) $\because$ ABCD is a parallelogram.

$\therefore$ $DC \parallel AB \Rightarrow DQ \parallel AB$

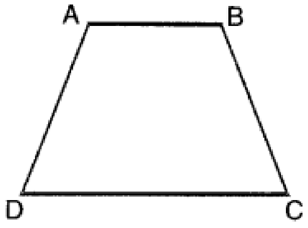
Now, in ΔARB, D is a mid-point of AR and $DQ \parallel AB$

$\therefore$ Q is a mid-point of BR [converse of mid-point theorem]

$\Rightarrow$ $BR = 2BQ$

Question.34

ABCD is a trapezium in which AB || CD and AD = BC (see fig.). Show that :



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) *diagonal AC = diagonal BD.* *[CBSE March 2011]*

Solution.

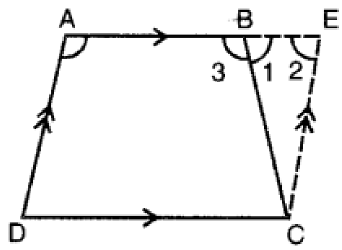
Given : ABCD is a trapezium, in which AB || DC and AD = BC.

To Prove : (*i*) $\angle A = \angle B$

(*ii*) $\angle C = \angle D$

(*iii*) $\Delta ABC \cong \Delta BAD$

(*iv*) diagonal AC = diagonal BD.



Const. : Produce AB to E, such that a line through C parallel to DA (CE || DA), intersects AB in E.

Proof : (*i*) In quad. AECD

AE || DC [given]

and AD || EC [by const.]

$\Rightarrow$ AECD is a parallelogram.

$\Rightarrow$ $AD = EC$ [opposite sides of $||^{gm}$]

But, $AD = BC$ [given]

$\Rightarrow$ $BC = EC$

$\Rightarrow$ $\angle 2 = \angle 1$ [angles opp. to equal sides of a Δ]

Also, $\angle 1 + \angle 3 = 180^\circ$ [linear pair]

and $\angle A + \angle 2 = 180^\circ$ [consecutive interior $\angle$s]

$\Rightarrow$ $\angle A + \angle 2 = \angle 1 + \angle 3$

$\Rightarrow$ $\angle A = \angle 3$ [$\because \angle 2 = \angle 1$]

or $\angle A = \angle B$

(*ii*) AB || DC and AD is a transversal.

$\therefore$ $\angle A + \angle D = 180^\circ$ [consecutive interior $\angle$s]

$\Rightarrow$ $\angle B + \angle D = 180^\circ$ ...(*i*)

[$\because \angle A = \angle B$]

Again, AB || DC and BC is a transversal.

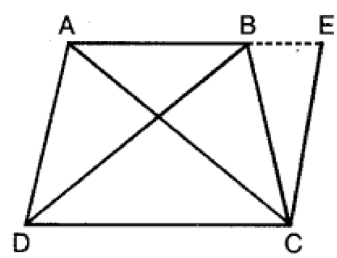
$\therefore$ $\angle B + \angle C = 180^\circ$ ...(*ii*)

From (*i*) and (*ii*), we have

$\angle B + \angle C = \angle B + \angle D$

$\Rightarrow$ $\angle C = \angle D$

(*iii*) In ΔABC and ΔBAD, we have



$AB = AB$ [common]

$BC = AD$ [given]

$\angle ABC = \angle BAD$ [proved above]

$\Rightarrow$ $\Delta ABC \cong \Delta BAD$ [by SAS congruence axiom]

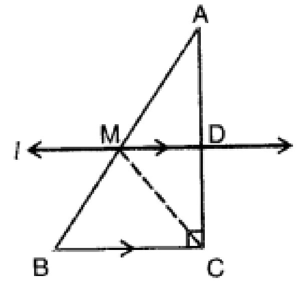
(*iv*) $\Rightarrow$ $AC = BD$ [c.p.c.t.]

Thus, diagonal AC = diagonal BD.

Question.35 ABC is a triangle right-angled at C. A line through the mid-point M of hypotenuse AB parallel to BC intersects AC at D. Show that:

(i) D is the mid-point of AC
(ii) MD⊥ AC
(iii) CM = MA =1/2 AB. [CBSE March 2012]

Solution.



Given : A right ΔABC, right-angled at C. A line through the mid-point M of hypotenuse AB parallel to BC intersects AC at D.

To Prove : (i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$

Proof : Since M is the mid-point of hyp. AB and MD || BC.

$\Rightarrow$ D is the mid-point of AC.

Since $\angle BCA = 90^\circ$ [given]

and $MD \parallel BC$ [corresp. ∠s]

$\Rightarrow$ $\angle MDA = \angle BCA = 90^\circ$

$\Rightarrow$ $MD \perp AC$

Now, in ΔADM and ΔCDM

$MD = MD$ [common]

$\angle MDA = \angle MDC$ [each = 90°]

$AD = CD$ [∵ D is the mid-point of AC]

$\Rightarrow$ $\Delta ADM \cong \Delta CDM$ [by SAS congruence axiom]

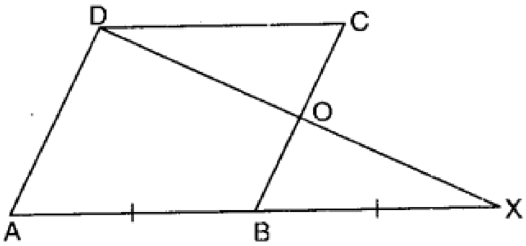
$\Rightarrow$ $AM = CM$ [c.p.c.t.]

Also, M is the mid-point of AB [given]

$\Rightarrow$ $CM = MA = \frac{1}{2}AB.$

Question.36

ABCD is a parallelogram and AB is produced to X such that AB = BX as shown in the figure. Show that DX and BC bisect each other at O. [CBSE-14-GDQNI3W]



Solution.

Here,	$AB = BX$	[given]
Also,	$AB = CD$	[opp. sides of the parallelogram]
$\therefore$	$BX = CD$	

Now, in ΔBOX and ΔCOD, we have

$\angle XBO = \angle DCO$	[alt. int. $\angle$s]
$\angle BXO = \angle CDO$	[alt. int. $\angle$s]
$BX = CD$	[proved above]

So, by using AAS congruence axiom,

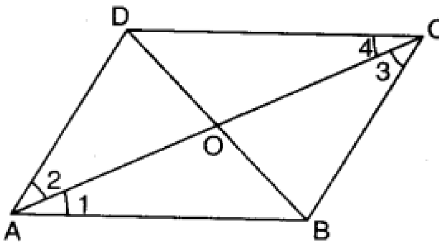
$$\Delta BOX \cong \Delta COD$$

$BO = CO$ and $OX = OD$ [c.p.c.t.]

Hence, DX and BC bisect each other at O.

Question.37 ABCD is a rhombus. Show that diagonals AC bisects ∠A as well as ∠C and diagonal BD bisects∠B as well as ∠D

Solution.



Since AB || DC and AC is a transversal.

$\therefore$ $\angle 1 = \angle 4$ ....(*i*) [alt. int. $\angle$s]

Similarly, AB || BC and AC is a transversal.

$\therefore$ $\angle 2 = \angle 3$ ....(*ii*) [alt. int. $\angle$s]

In ΔABC

$AB = BC$ [$\because$ sides of rhombus are equal]

$\therefore$ $\angle 3 = \angle 1$ ....(*iii*) [$\angle$s opp. to equal sides of a Δ]

From (*i*), (*ii*) and (*iii*), we obtain

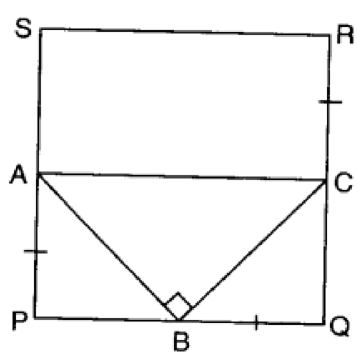
$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Thus, AC bisects $\angle A$ as well as $\angle C$.

Similarly, we can prove that BD bisects $\angle B$ as well as $\angle D$.

Question.38

PQRS is a square and $\angle ABC = 90°$ as shown in the figure. If $AP = BQ = CR$, then prove that $\angle BAC = 45°$. [CBSE-14-GDQNI3W]



Solution.

Since PQRS is a square.

$\therefore \quad PQ = QR \quad \ldots (i) \quad$ [$\because$ sides of a square are equal]

Also, $\quad BQ = CR \quad \ldots (ii) \quad$ [given]

Subtracting (*ii*) from (*i*), we obtain

$$PQ - BQ = QR - CR$$

$\Rightarrow \quad PB = QC \quad \ldots (iii)$

In ΔAPB and ΔBQC

$AP = BQ \quad$ [given]

$\angle APB = \angle BQC = 90^\circ \quad$ [each angle of a square is 90°]

$PB = QC \quad$ [using (*iii*)]

So, by using SAS congruence axiom, we have

$$\Delta APB \cong \Delta BQC$$

$\therefore \quad AB = BC \quad$ [c.p.c.t.]

Now, in ΔABC

$AB = BC \quad$ [proved above]

$\therefore \quad \angle ACB = \angle BAC = x^\circ$ (say) $\quad$ [$\angle$s opp. to equal sides]

Also, $\quad \angle B + \angle ACB + \angle BAC = 180^\circ$

$\Rightarrow \quad 90^\circ + x^\circ + x^\circ = 180^\circ$

$\Rightarrow \quad 2x^\circ = 90^\circ$

$\Rightarrow \quad x^\circ = 45^\circ$

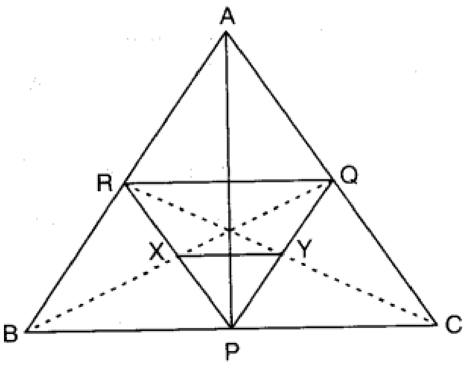
Hence, $\quad \angle BAC = 45^\circ$

Question.39

Hence, $\quad \angle BAC = 45^\circ$

In the figure, P, Q and R are the mid-points of the sides BC, AC and AB of ΔABC. If BQ and PR intersect at X and CR and PQ intersect at Y, then show that $XY = \frac{1}{4}BC$.

[CBSE-14-ERFKZ8H]



Solution. Here, in AABC, R and Q are the mid-points of AB and AC respectively.

$\therefore$ By using mid-point theorem, we have

$$RQ \parallel BC \text{ and } RQ = \frac{1}{2}BC$$

$\therefore$ $RQ = BP = PC$ [$\because$ P is the mid-point of BC]

$\therefore$ $RQ \parallel BP$ and $RQ \parallel PC$

In quadrilateral BPQR

$RQ \parallel BP$, $RQ = BP$ [prove above]

$\therefore$ BPQR is a parallelogram [$\because$ one pair of opp. sides is parallel as well as equal]

$\therefore$ X is the mid-point of PR [$\because$ diagonals of a $\|^{gm}$ bisect each other]

Now, in quadrilateral PCQR

$RQ \parallel PC$ and $RQ = PC$ [proved above]

$\therefore$ PCQR is a parallelogram [$\because$ one pair of opp. sides is parallel as well as equal]

$\therefore$ Y is the mid-point of PQ [$\because$ diagonals of a $\|^{gm}$ bisect each other]

In ΔPQR

X and Y are mid-points of PR and PQ respectively.

$\therefore$ $XY \parallel RQ$ and $XY = \frac{1}{2}RQ$ [by using mid-point theorem]

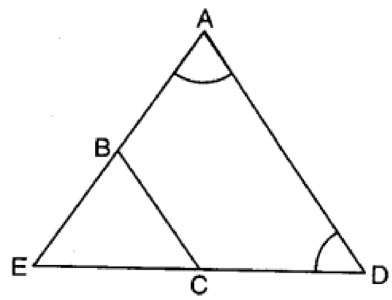
$$XY = \frac{1}{2}\left(\frac{1}{2}BC\right) \quad \left[\because RQ = \frac{1}{2}BC\right]$$

$$\Rightarrow \quad XY = \frac{1}{4}BC$$

Question.40

In the given figure, AE = DE and BC || AD. Prove that the points A, B, C and D are concyclic. Also, prove that the diagonals of the quadrilateral ABCD are equal.

[CBSE-14-ERFKZ8H]



Solution.

Since $AE = DE$

$\therefore$ $\angle D = \angle A$ .... (*i*) [$\because$ $\angle$s opp. to equal sides of a Δ]

Again, $BC \parallel AD$

$\therefore$ $\angle EBC = \angle A$ .... (*ii*) [corresponding $\angle$s]

From (*i*) and (*ii*), we have

$\angle D = \angle EBC$ .... (*iii*)

But $\angle EBC + \angle ABC = 180^\circ$ [a linear pair]

$\Rightarrow$ $\angle D + \angle ABC = 180^\circ$ [using (*iii*)]

Now, a pair of opposite angles of quadrilateral ABCD is supplementary.

Thus, ABCD is a cyclic quadrilateral *i.e.*, A, B, C and D are concyclic.

In ΔABD and ΔDCA

$\angle ABD = \angle ACD$ [$\angle$s in the same segment for cyclic quad. ABCD]

$\angle BAD = \angle CDA$ [using (*i*)]

$AD = AD$ [common]

So, by using AAS congruence axiom, we have

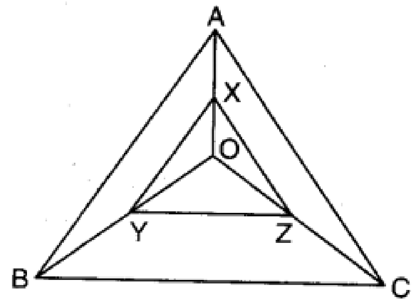
$\Delta ABD \cong \Delta DCA$

Hence, $BD = CA$ [c.p.c.t.]

Question.41

In ΔABC, AB = 8cm, BC = 9cm and AC = 10 cm. X, Y and Z are mid-points of AO, BO and CO respectively as shown in the figure. Find the lengths of the sides of ΔXYZ.

[CBSE-15-6DWMW5A]



Solution.

Here, in ΔABC, AB = 8 cm, BC = 9 cm, AC = 10 cm.

In ΔAOB, X and Y are the mid-points of AO and BO.

$\therefore$ By using mid-point theorem, we have

$$XY = \frac{1}{2}AB = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$$

Similarly, in ΔBOC, Y and Z are the mid-points of BO and CO.

$\therefore$ By using mid-point theorem, we have

$$YZ = \frac{1}{2}BC = \frac{1}{2} \times 9 \text{ cm} = 4.5 \text{ cm}$$

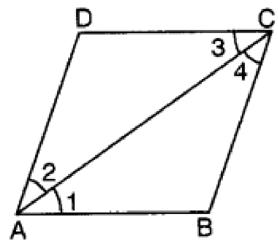
And, in ΔCOA, Z and X are the mid-points of CO and AO.

$$\therefore \quad ZX = \frac{1}{2}AC = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$$

Hence, the lengths of the sides of ΔXYZ are XY = 4 cm, YZ = 4.5 cm and ZX = 5 cm.

Question. 42 ABCD is a parallelogram in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that ABCD is a rhombus. [CBSE-14-17DIG1U]

Solution.



Since ABCD is parallelogram

$\therefore$ AB || DC

Now, AB || DC and AC is a transversal

$\therefore$ $\angle 1 = \angle 3$ .... (*i*) [alt. int. $\angle$s]

Again, AD || BC and AC is a transversal

$\therefore$ $\angle 2 = \angle 4$ .... (*ii*) [alt. in $\angle$s]

Also, AC bisects $\angle A$

$\therefore$ $\angle 1 = \angle 2$ .... (*iii*)

From (*i*), (*ii*) and (*iii*), we have

$\angle 3 = \angle 4$

Thus, AC bisects $\angle C$

From (*ii*) and (*iii*), we have

$\angle 1 = \angle 4$

$\Rightarrow$ BC = AB [sides opp. to equal $\angle$s of a Δ]

But AB = DC and BC = AD [$\because$ ABCD is a parallelogram]

$\Rightarrow$ AB = BC = CD = DA

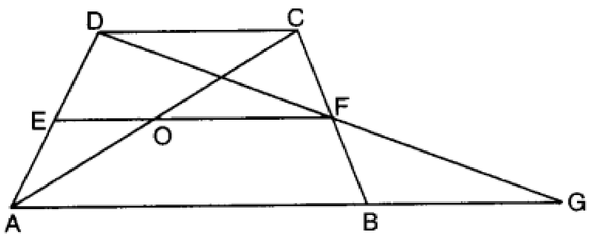
Hence. ABCD is a rhombus.

Question. 43

In the figure, ABCD is a trapezium in which AB || DC. E and F are the mid-points of AD and BC respectively. DF and AB are produced to meet at G. Also, AC and EF intersect at the point O. Show that :

(i) EO || AB (ii) AO = CO

[CBSE-14-17DIG1U]



Solution.

Here, E and F are the mid-points of AD and BC respectively.

In ΔBFG and ΔCFD

$BF = CF$ [given]

$\angle BFG = \angle CFD$ [vert. opp. $\angle s$]

$\angle BGF = \angle CDF$ [alt. int. $\angle s$, as $AB \parallel DC$]

So, by using AAS congruence axiom, we have

$\Delta BFG \cong \Delta CFD$

$\Rightarrow$ $DF = FG$ [c.p.c.t.]

Now, in ΔAGD, E and F are the mid-points of AD and GD.

$\therefore$ By mid-point theorem, we have

$EF \parallel AG$

or $EO \parallel AB$

Also, in ΔADC, $EO \parallel DC$

$\therefore$ EO is a line segment from mid-point of one side parallel to another side.

Thus, it bisects the third side.

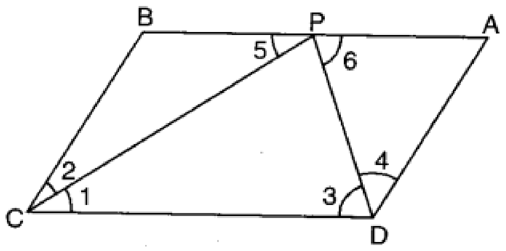
Hence, $AO = CO$

Question. 44 ABCD is a parallelogram. If the bisectors DP and CP of angles D and C meet at P on side AB, then show that P is the mid-point of side AB. [CBSE-15-NS72LP7]

Solution.

Since DP and CP are angle bisectors of $\angle D$ and $\angle C$ respectively.

$\therefore$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$



Now, $AB \parallel DC$ and CP is a transversal.

$\therefore$ $\angle 5 = \angle 1$ [alt. int. $\angle s$]

But $\angle 1 = \angle 2$ [given]

$\therefore$ $\angle 5 = \angle 2$

Now, in ΔBCP, $\angle 5 = \angle 2$

$\Rightarrow$ $BC = BP$ ... (*i*) [sides opp. to equal $\angle s$ of a Δ]

Again, $AB \parallel DC$ and DP is a transversal.

$\therefore$ $\angle 6 = \angle 3$ [alt. int. $\angle s$]

But $\angle 4 = \angle 3$ [given]

$\therefore$ $\angle 6 = \angle 4$

Now, in ΔADP, $\angle 6 = \angle 4$

$\Rightarrow$ $DA = AP$ .... (*ii*) [sides opp. to equal $\angle s$ of a Δ]

Also, $BC = DA$ ... (*iii*) [opp. sides of parallelogram]

From (*i*), (*ii*) and (*iii*), we have

$BP = AP$

Hence, P is the mid-point of side AB.

Value Based Questions (Solved)

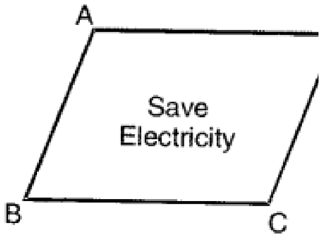
Question.1

Ankush prepare a poster in the form of parallelogram, as in figure.

(i) If $\angle A = (5x + 7)^\circ$ and $\angle B = (3x - 3)^\circ$, find all the angles of parallelogram ABCD.

(ii) Which mathematical concept is used in this question ?

(iii) By writing a slogan on poster which value is depicted by Ankush ?



Solution.

(i) Since sum of adjacent angles of a parallelogram is 180°.

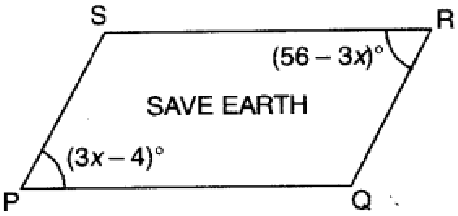
$$\therefore \quad \text{We have } \angle A + \angle B = 180^\circ$$
$$\Rightarrow \quad 5x + 7 + 3x - 3 = 180^\circ$$
$$\Rightarrow \quad 8x + 4 = 180^\circ$$
$$\Rightarrow \quad 8x = 176^\circ$$
$$\Rightarrow \quad x = \frac{176^\circ}{8} = 22^\circ$$
$$\therefore \quad \angle A = (5x + 7)^\circ$$
$$= 5 \times 22^\circ + 7 = 117^\circ$$
$$\angle B = (3x - 3)^\circ = 3 \times 22^\circ - 3 = 63^\circ$$
$$\angle C = \angle A = 117^\circ \quad [\text{opposite angles of a } \|^{gm}]$$
$$\text{and} \quad \angle D = \angle B = 63^\circ$$

(ii) Properties of parallelogram.

(iii) By saving electricity, saving energy.

Question.2

In the given figure, two opposite angles of a parallelogram PQRS are $(3x - 4)^\circ$ and $(56 - 3x)^\circ$. Find all the angles of given parallelogram. What value depicted by the given solgan ?



Solution.

We know that opposite angles of a parallelogram are equal.

$\therefore \quad \angle P = \angle R$

$\Rightarrow \quad 3x - 4 = 56 - 3x$

$\Rightarrow \quad 6x = 60$

$\Rightarrow \quad x = 10$

Thus, $\quad \angle P = \angle R = 3 \times 10 - 4 = 26^\circ$

Also, $\quad \angle P + \angle Q = 180^\circ$ [adjacent angles of a $\|^{gm}$]

$\Rightarrow \quad \angle Q = 180^\circ - 26^\circ = 154^\circ$

Thus, $\quad \angle Q = \angle S = 154^\circ$

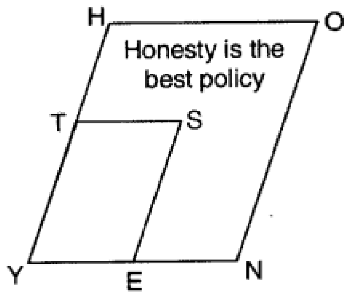
Hence, the four angles of the parallelogram PQRS are 26°, 154°, 26° and 154°.

We should care our earth to have good eco balance.

Question.3

In the given figure TSEY and HONY are two parallelograms. If $\angle HON = 47^\circ$, then find the measure of $\angle HYN$, $\angle TSE$ and $\angle YNO$.

Write the role of value honesty in the field of sports.



Solution.

Here, in parallelogram HONY, $\angle HON = 47^\circ$

We know that opposite angles of a parallelogram are equal.

$\therefore \quad \angle HYN = \angle HON = 47^\circ$

Now, in $\|^{gm}$ TSEY, we have

$\angle TSE = \angle HYN$ or $\angle TYE = 47^\circ$

Also, $\angle HYN + \angle YNO = 180^\circ$

$\Rightarrow \quad 47^\circ + \angle YNO = 180^\circ$

$\Rightarrow \quad \angle YNO = 180^\circ - 47^\circ = 133^\circ$

Thus, $\quad \angle HYN = 47^\circ$, $\angle TSE = 47^\circ$ and $\angle YNO = 133^\circ$

Honesty unites the team-mates and provide cooperation, a source of inspiration (team-spirit with sportsmanship).